Directional emission from photonic crystal waveguide terminations using particle swarm optimization

M. Sathish Kumar, Sergey Menabde, Sunkyu Yu, and Namkyoo Park

Photonic Systems Laboratory, School of EECS, Seoul National University, Gwanak Ku, Seoul 151 742, South Korea

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We report particle swarm optimization (PSO) of photonic crystal (PC) structures through an example of a PC waveguide termination to achieve directional emission. PC waveguide termination is optimized by using the PSO algorithm by evaluating a fitness function by the scattering matrix method. We consider two different structures, reported earlier and designed through intuition and trial and error, for our optimization and compare the results obtained. Our results show that optimizing with PSO can provide more direct and intense beams compared with designs based purely on intuition and trial and error. Compared with the two earlier reported directional emission PC structures, increases in intensity by factors of 1.25 and 2 and decreases in beam divergence by factors of 2.5 and 2.4, respectively, were achieved. © 2010 Optical Society of America

1. INTRODUCTION

Photonic crystals (PCs) and devices based on them have been the subject of active research investigations for the past few years. The single greatest driving factor behind these investigations has been the promise they hold for miniaturization of photonic circuits, which could eventually lead to a full-fledged realization of nanophotonic systems. Meanwhile, though the physics of PCs have been established in fundamental principles, in practice the design and structure optimization of PC devices requires a deeper understanding of the PC as well as the underlying device theory. This results in most cases in an intensive reiteration procedure to reach the final design [1–6].

One such problem of practical importance that poses great difficulties in realizing high-performance, fabrication-friendly design is the PC waveguide termination to achieve highly directive emission. Several approaches have been proposed to solve this problem. However, these solutions were either suboptimum, in that the design was based more on intuition and trial and error without any genuine optimization approach [1–4], or were optimized to give the best possible performance at the cost of fabrication complexities [5,6], making it hard to realize such optimized structures with the existing state-of-the-art fabrication technologies.

Though the structures for directional emission reported in [1–4] are far easier to fabricate than the topology-optimized structures reported in [5,6], the question still remains as to whether the structures reported in [1–4], based on an optimization that is guided by intuition and trial and error, could achieve solutions or structures near the theoretical limit in directing or focusing light from PC waveguides. In addition, it may be noted that the optimization of directional emission from PC waveguides is a problem of nonanalytic nature with an associated configuration space of huge dimensions. In this context, it would be of much more practical use and interest to explore the possibilities of achieving such optimized structures that can ensure a near-theoretical limit performance and, at the same time, retain the relative fabrication ease of the structures reported in [1–4].

One of the approaches that can be taken for the optimization of multidimensional problems of this kind, especially in the domain of computational electromagnetism, is particle swarm optimization (PSO) [7]. In the following, we use the term “analog PSO” to mean that PSO algorithm in which the solution space is considered to be continuous, unlike in Boolean PSO, where the solution space is discrete. In this paper, the term PSO will refer to analog PSO. In this paper, we use PSO for optimizing PC structures and compare its performance with Boolean PSO as well as with results previously reported that were based purely on intuitive design. As hinted above, we do this by taking the optimization problem of directional emission from a PC waveguide as an example. We find that the PSO-optimized structures could provide better directivity and intensity over structures designed purely based on intuition [1,2]. More specifically, our results show that optimization with PSO can provide a reduction in the beam FWHM from 10° to 8° as well as provide an increase by a factor of 1.25 in output intensity compared with the surface-mode-assisted structure [1], while compared with the tapered design approach proposed in [2], our results provide a reduction in FWHM from 26° to 11° and an increase in intensity by a factor of 2.

2. BRIEF REVIEW OF PARTICLE SWARM OPTIMIZATION

PSO is a novel stochastic evolutionary optimization technique based on the movement and intelligence of swarms
proposed first by Kennedy and Eberhert [7]. The method has been demonstrated to be superior to the well-known genetic algorithm for certain difficult optimization problems [8]. Further, compared with the genetic algorithm, PSO is easier to implement and has fewer parameters to control. Discrete versions of PSO were also proposed in [9, 10]. Though PSO is still in its developmental stages, it has already given some promising results in the domain of photonics in particular and electromagnetics in general [10–15].

In PSO, a set of particles are initiated at random positions in the solution space comprising the entire set of possible solutions. A fitness function is used to guide these particles through the solution space to that position where the fitness function has its target value. The whole process can be expressed by means of a simple set of equations as given below:

$$v_d^i(n + 1) = w \cdot v_d^i(n) + c_1 \cdot r_1^i(n)(p_d^i(n) - x_d^i(n)) + c_2 \cdot r_2^i(n)(g_d(n) - x_d^i(n)),$$

(1)

$$x_d^i(n + 1) = x_d^i(n) + v_d^i(n + 1),$$

(2)

where $v_d^i(n)$ is the velocity of the $i$th particle along the $d$th coordinate of the solution space after the $n$th iteration and $x_d^i(n)$ is the $d$th coordinate of the $i$th particle after the $n$th iteration. The term $p_d^i(n)$ refers to the $d$th coordinate of the best position encountered by the $i$th particle until the most recent iteration and is identified as the personal best position. A “best position” is that where the fitness function has a value that is closest to its final desired value. The term $g_d(n)$ is the $d$th coordinate of the global best position, which is the best among the personal best positions of all the particles in the swarm until the most recent iteration. Note that the global best position does not have the superscript identifying the particle. The terms $c_1$ and $c_2$ are constants identified as the cognitive and social rates, respectively, since they determine the extent by which the personal best position and the global best position influence the movement of the particle. The terms $r_1^i$ and $r_2^i$ are uniformly distributed random numbers in the range 0–1 and account for the randomness in the swarm movement. Finally, the term $w$ is a constant that acts as the inertia of the particle in that it determines the extent by which the velocity after the $n$th iteration is affected by the velocity after the $(n-1)$th iteration. As the iteration count increases, progressively each particle in the swarm will be guided to that position where the fitness function has its desired value. A complete account of PSO can be found in [12]. When the solution space is discrete, Boolean PSO is preferred, and Eqs. (1) and (2) given above become altered with additions and products replaced by exclusive OR and AND operations. A full account of Boolean PSO can be found in [10].

3. IMPLEMENTATION OF PARTICLE SWARM OPTIMIZATION FOR PHOTONIC CRYSTAL STRUCTURES

As mentioned above, for application of PSO to PC structures, we consider directional emission from a PC waveguide termination. Figure 1 shows the two-dimensional PC waveguide under consideration to be optimized for directional emission. The cylindrical rods have a diameter $d$ and are of permittivity $\varepsilon_1$, and the background region is free space with a permittivity $\varepsilon_2=1$. The lattice constant of defectless crystal is $a$, and the normalized operating frequency is expressed as $a/\lambda$. The propagation is assumed to be TM (electric field polarized in the $y$ direction). We define the task of optimization as the maximization of power flow through a target plane at a distance of $D\alpha$ within an angle of $\theta$ degrees defined from the waveguide exit (Fig. 1). It is worth noting that the PSO technique can as well be applied to any other PC structure optimization once the proper optimization goal has been identified. An example can be found in [14].

The first example to be considered has $\varepsilon_1=11.56$, $d=0.36a$, and $a/\lambda=0.38$. It can be seen from Fig. 2 that for these parameters the waveguide is indeed single mode. To achieve directional emission for the current example, in [16], the diameter of rods composing the outer-most column on either side of the waveguide was reduced so as to excite a surface mode. Further, corrugations were formed on this column by displacing the even-numbered

Fig. 1. (Color online) Waveguide structure to be optimized; the power at the target plane is to be maximized.

Fig. 2. (Color online) Dispersion diagram for the waveguide under consideration.
rods (numbered consecutively away from the waveguide) from their lattice sites by pulling them into the crystal volume (Fig. 2 of [16]). This resulted in radiation of the excited surface mode, which combined with the waveguide output to provide a directional beam in the far field.

The directional output so obtained was further improved considerably in [1] by altering the diameter of the rods immediately behind the outermost column of rods and by altering the permittivity of the rods in the outermost column. Moreover, in [1], the corrugations were formed not as in [16] but by displacing the odd-numbered rods from their lattice sites by pushing them away from the crystal volume. In this paper, we identify such structures, providing directional emission through creation of surface modes and radiating them via corrugations, as zigzag structures.

We note that there is scope for further improvement in the directional emission of zigzag structures, since no use of a genuine optimization algorithm was made in [1,16] to decide the best possible combination of the rod diameters of the two outermost columns, the relative positioning of the rods in the outermost column to create corrugations, and the permittivity of the rods in the outermost column. Toward this, we identify five parameters as follows: (1) \( \text{Pull} \), the displacement of the even numbered rods into the crystal volume; (2) \( \text{Push} \), the displacement of the odd numbered rods away from the crystal volume; (3) and (4) diameters of the rods comprising the corrugated surface layer and the layer just behind the corrugated surface layer, identified as \( d_e \) and \( d_{cb} \), respectively; and (5) the permittivity of the rods composing the corrugated surface layer identified as \( \varepsilon_{surf} \). To facilitate a better understanding of what the above parameters actually are, for the structure reported in [1], these parameters would be, \( \text{Pull}=0 \), \( \text{Push}=0.4a \), \( d_e=0.18a \), \( d_{cb}=0.27a \), and \( \varepsilon_{surf}=12.96 \), while for the structure reported in [16], these parameters would be 0.3\( a \), 0, 0.18\( a \), 0.36\( a \), and 11.56, respectively. With these five parameters to optimize, we have a five-dimensional solution space that can be searched for the optimum solution by using the PSO. Since the optimization exercise is to have a directional emission from the PC waveguide termination, we choose the fitness function as the power through the target plane located at the distance \( Da \) from the waveguide output (Fig. 1). The maximization of power through this target plane is the goal of optimization.

Figure 3 shows the optimized structure obtained for the above waveguide structure and parameters as well as the electric field distribution within the optimized structure. For the PSO optimized zigzag structure, we chose to have nine corrugations on the surface layer [1,16]. The target plane was assumed to be at a distance of 35\( a \) (\( D=35 \) in Fig. 1) from the waveguide exit, and the angle \( \theta \) was set at 4\( ^\circ \). All parameters relevant to PSO have been selected from Table II of [12]. A plot of the normalized intensity along a radius of 35\( a \) from the waveguide exit for the PSO optimized zigzag structure and the zigzag structure reported in [1] is also shown in Fig. 3. As can be clearly observed from the field and normalized intensity plots, the PSO optimized zigzag structure provides better directional emission as well as intensity. More specifically, it is 1.25 times more intense with a FWHM reducing from 10\( ^\circ \) to 8\( ^\circ \) compared with the structure reported in [1]. For the PSO zigzag structure, it was also found that the z-directed Poynting vector before and after the surface layer was 1.3 times greater than that for the structure reported in [1] and also had a well defined sinusoidal characteristic. It is worth mentioning here that the sensitivity of the directional emission on changes in the operating frequency for the above PSO structure was also investigated. It was observed that the directional emission characteristic was retained over a wide frequency range. Figure 3(d) reflects this, wherein the angular distribution of intensity is shown for normalized frequencies 0.37 as well as 0.39 over and above 0.38, which is the normalized frequency for which the optimization was carried out.

It is worth mentioning here that optimization based on Boolean PSO was also attempted for the present problem. A region of 18 rows and 2 columns of rods on either sides of the waveguide exit was selected, and it was determined through Boolean PSO which of the rods among these \( 2 \times (18 \times 2) \) rods needed to be retained or removed in order to have maximum power through the target plane (Fig. 1). It may be noted that because of the symmetry of the problem, though there are \( 2 \times (18 \times 2) \) rods to be considered, only 18 \( \times 2 \) rods on any one side of the waveguide need be considered. Thus the Boolean PSO needs to search through a discrete solution space that has \( 2^{36} \) possible solutions in it. The results obtained through Boolean PSO were found to be inferior to both the PSO zigzag structure as well as the structure reported in [1] and hence are not included in Fig. 3.

We consider a second example, the structure reported in [2] with \( \varepsilon_1=10.24 \), \( d=0.44a \), and \( a/\lambda \) as 0.38. A dispersion diagram similar to Fig. 2 can be obtained for this waveguide structure, and it can be seen that the waveguide is indeed single mode for the above set of parameters. The structure about to be considered was optimized in [2] through a simple and intuitive removal of rods at the waveguide exit, resulting in a tapered termination of the waveguide. The spatial thickness of the waveguide termination was altered to result in interference patterns under certain conditions, which resulted in directional beam patterns [2]. In this case, for optimization of the design with PSO, we consider a region comprising three rows and six columns on either side of the waveguide and move the rods at positions (2,3) to (2,6) and (3,2) to (3,6). This results in the tapered termination reported in [2]. Note that the row count ends just above the waveguide and the column count ends at the waveguide exit for the 3 \( \times 6 \) region above the waveguide. Below the waveguide, it is a mirror image of this. We apply PSO for optimizing this tapered termination in two different ways, resulting in two different PSO optimized tapered termination structures. The first approach is to vary both the diameter as well as the vertical \( x \) direction (Fig. 1) displacement of the nine remaining rods in the 3 \( \times 6 \) region independently, while the second approach is to vary the vertical displacement of just the six rods at (3, 1), (2, 2), and (1, 3)–(1,6) while retaining their diameters. Thus for the first approach (tapered PSO-1) there are a total of 18 parameters to vary; the diameters as well as the vertical displacements of the 9 remaining rods in the 3 \( \times 6 \) region. For the second approach (tapered PSO-2) there are a total
of seven parameters to vary; the vertical displacement of the six rods and the diameter of the rods, which is held the same for all six rods. For Boolean PSO, we choose a region comprising eight rows and two columns on either side of the waveguide exit, and the approach to apply Boolean PSO to the present case remains the same as that in the previous example.

Figure 4 shows the optimized structures with PSO as well as Boolean PSO and the electric field distributions within them. A plot of the normalized intensity along a radius of 10a from the waveguide exit for the PSO and Boolean PSO optimized waveguides as well as the tapered termination structure reported in [6] is also shown in Fig. 4. Table 1 provides the array of rod diameter and displacements within the optimized region. It can be seen that though tapered PSO-1 gives a better performance than tapered PSO-2, the extremely thin rod at (1,3) with a diameter of 0.24 × 0.44a can make the fabrication task difficult. It can also be seen that the Boolean PSO structure gives the best performance out of the four; it is 2 times more intense with a FWHM reducing from 26° to 11° compared with the structure reported in [2]. Tapered PSO-1 and tapered PSO-2, respectively, provide an increase in intensity over the structure reported in [2] by factors of 1.8 and 1.34 with FWHM 17° and 24°. For the PSO tapered structures, the improved performance is due to the gradual, adiabatic variation in the taper. By creating surface modes for the case considered in Fig. 4 through Boolean PSO, even better performance, as shown in Figs. 4(f) and 4(g) could be achieved. In a way, this result shows that the approach of simple removal and retaining of rods and thus creating a tapered termination is inferior to the approach of creating surface modes and radiating them via corrugations as in the zigzag structure. With regard to the frequency dependence of the PSO structures shown in Fig. 4, a behavior similar to that in the zigzag PSO struc-
ture was observed. We did attempt a Boolean PSO optimization for the present example with a $4 \times 4$ optimization region but ended up getting structures that had cavities in the optimization region. Though such structures had high directivity and intensity compared with the tapered termination in [2], their bandwidth was found...
Table 1. Diameter and Displacement of Rods within the Optimized Region Normalized with Respect to $d$ and $a$, Respectively

<table>
<thead>
<tr>
<th>Tapered PSO-1</th>
<th>Column</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Row</strong></td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>0.75 (+0.09)</td>
</tr>
<tr>
<td>2</td>
<td>0.74 (−0.21)</td>
</tr>
<tr>
<td>3</td>
<td>0.93 (+0.29)</td>
</tr>
<tr>
<td><strong>Tapered PSO-2</strong></td>
<td></td>
</tr>
<tr>
<td><strong>Row</strong></td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>1 (0)</td>
</tr>
<tr>
<td>2</td>
<td>1 (0)</td>
</tr>
<tr>
<td>3</td>
<td>0.81 (+0.02)</td>
</tr>
</tbody>
</table>

*Positive displacement is away from the waveguide.

4. CONCLUSIONS

We have demonstrated the optimization of PC structure by using analog and Boolean PSOs through the example of waveguide terminations for directional emission. Our results show that the FWHM as well as the intensity at the target plane improves considerably compared with structures designed through only intuition and/or trial and error. We also observe that analog PSO is better than Boolean PSO for the zigzag structure, since analog PSO can search through a more inclusive solution space to come out with optimum solutions. On the other hand, a tapered termination design optimization using analog PSO seemed to give inferior performance compared with Boolean PSO using an $8 \times 2$ optimization region. We attribute this to the inherent inferiority of the tapered termination design compared with achieving directional emission by using a surface-mode-type design. The structures obtained are relatively easier to fabricate compared with the others proposed based on topology optimization. Though we have demonstrated our results based on the scattering matrix method, the PSO can be applied as well to PC structures modeled through other means, for example, the finite-difference time domain. With the inherent property of PSO providing a near-optimal solution with its robust capability to search through the solution space as demonstrated in this paper, we believe that our results will permit more systematic optimization approaches for the design of PC devices of other functionalities.

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