Extraordinary Magnetic Field Enhancement with Metallic Nanowire: Role of Surface Impedance in Babinet’s Principle for Sub-Skin-Depth Regime

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We propose and analyze the “complementary” structure of a metallic nanogap, namely, the metallic nanowire for magnetic field enhancement. A huge enhancement of the field up to a factor of 300 was achieved. Introducing the surface impedance concept, we also develop and numerically confirm a new analytic theory which successfully predicts the field enhancement factors for metal nanostructures. Compared to the predictions of the classical Babinet principle applied to a nanogap, an order of magnitude difference in the field enhancement factor was observed for the sub-skin-depth regime nanowire.

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It is well known that nanosized metallic structures such as nanogaps can achieve huge electric field enhancement through the phenomena of extraordinary transmission (EOT) [1–11]. Recent developments in EOT include the demonstration of nonresonant, factor of 1000 enhancements in the electric field, through a demonstration of nonresonant, factor of 1000 enhancement of electric field in the sub-skin-depth regime. In sharp contrast to the predictions of the classical Babinet principle applied to a nanogap, an order of magnitude difference in the field enhancement factor was observed for the sub-skin-depth regime nanowire.

The field \( H_z(E_z) \) for the TM (TE) wave in free space for 2D problems can be expressed as below:

\[
\frac{\partial^2 H_z(E_z)}{\partial x^2} + \frac{\partial^2 H_z(E_z)}{\partial y^2} + k_0^2 H_z(E_z) = 0.
\]

A general solution of (1) can be expressed as a linear combination of plane waves. With the scattering objects of difference in the field enhancement were observed between nanogap and nanowire. Results show excellent agreement of the developed theory with the finite-difference time-domain (FDTD) analysis.

Figure 1 shows the system under consideration. Figure 1(a) is the nanogap and Fig. 1(b) is the nanowire. A plane wave of wavelength \( \lambda_0 \) impinges normally on these structures. The polarization considered is transverse magnetic (TM) for the nanogap and transverse electric (TE) for the nanowire. For the present, the thickness of the structure is assumed to be infinitesimal so as to isolate the effect of nonzero impedance of the real metal on the complementary scattering behavior. The effects of finite thickness on the scattering are discussed later. \( R_s \) stands for the surface resistivity in ohms per square. When \( R_s = 0 \), the metal becomes a perfect electric conductor (PEC).

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![FIG. 1. Diagram of the (a) nanogap and (b) nanowire structure having width \( w \) and infinitesimal thickness. \( R_s \) is the surface resistivity of the metal in ohms per square, and \( Z \) is the free space impedance. The structure is illuminated by the TM (TE) wave for the nanogap (nanowire) having wavelength \( \lambda_0 = 2\pi/k_0 \). The superscript (i) denotes incident fields.](Image 342x149 to 534x232)
at \( y = 0 \), following the procedures in [13], the scattered field components can be shown to be as
\[
H_z^{(s)} = \pm \int_C P(\cos \alpha) \exp[ik_0 r \cos(\theta - \alpha)] d\alpha \quad E_x^{(s)} = -Z \int_C \sin \alpha P(\cos \alpha) \exp[ik_0 r \cos(\theta - \alpha)] d\alpha
\]
\[
E_y^{(s)} = \pm Z \int_C \cos \alpha P(\cos \alpha) \exp[ik_0 r \cos(\theta - \alpha)] d\alpha,
\]
\[
E_z^{(s)} = \int_C P(\cos \alpha) \exp[ik_0 r \cos(\theta - \alpha)] d\alpha \quad H_x^{(s)} = \pm \frac{1}{Z} \int_C \sin \alpha P(\cos \alpha) \exp[ik_0 r \cos(\theta - \alpha)] d\alpha
\]
\[
H_y^{(s)} = -\frac{1}{Z} \int_C \cos \alpha P(\cos \alpha) \exp[ik_0 r \cos(\theta - \alpha)] d\alpha,
\]
respectively, for TM and TE incidence. Here \( k_0 \) and \( Z \) are the wave number and impedance, respectively, in free space. \( C \) is the path in the complex \( \alpha \) plane along which \( \cos \alpha \) ranges through real values from \(-\infty \) to \( \infty \) [13]. The superscript \( (s) \) denotes scattered components \( (E = E^{(s)} + E^{(i)}, H = H^{(s)} + H^{(i)}) \) and the upper sign is for \( y > 0 \) and the lower sign is for \( y < 0 \).

The boundary conditions for infinitesimally thin metals are well known [16] and so are the symmetry conditions on the scattered fields [13], as given below:
\[
\hat{n} \times [E(y = 0+) - E(y = 0-)] = 0 \quad \text{on air (}\hat{n} \times E(y = 0) = 0 \quad \text{on metal)}
\]
\[
\begin{align*}
H_x^{(s)}(y = 0+) = -H_x^{(s)}(y = 0-) \\
H_y^{(s)}(y = 0+) = H_y^{(s)}(y = 0-) \\
H_z^{(s)}(y = 0+) = -H_z^{(s)}(y = 0-) \\
E_x^{(s)}(y = 0+) = E_x^{(s)}(y = 0-) \\
E_y^{(s)}(y = 0+) = -E_y^{(s)}(y = 0-) \\
E_z^{(s)}(y = 0+) = E_z^{(s)}(y = 0-)
\end{align*}
\]

Here \( \hat{n} \) is the outward unit vector parallel to the \(+y \) direction and \( J_s \) is the surface current density. Now putting (2) and (3) into (4) and (5) we can arrive at the dual integral equations to be solved for \( P(\cos \alpha) \). At \( y = 0+ \), with \( \cos \alpha = \mu \), for nanogap (6) and nanowire (7) we get
\[
\int_{-\infty}^{\infty} \frac{P(\mu)}{1 - \mu^2} \exp(ik_0 x \mu) d\mu = 0 \quad \text{for} \quad \left| x \right| < \frac{w}{2}
\]
\[
\int_{-\infty}^{\infty} \left( \frac{2R_s}{\sqrt{1 - \mu^2}} + Z \right) P(\mu) \exp(ik_0 x \mu) d\mu = E^{(i)} \quad \text{for} \quad \left| x \right| > \frac{w}{2}
\]

Using the convolution theorem [17] from spectral analysis, we can express (6) and (7) as, for nanogap (8) and nanowire (9)
\[
\int_{-\infty}^{\infty} E_x^{(s)}(x') c(x-x') dx' = 0 \quad \text{for} \quad \left| x \right| < \frac{w}{2}
\]
\[
\frac{k_0}{2\pi} \int_{-\infty}^{\infty} H_x^{(s)}(x') c(x-x') dx' + 2\frac{R_s}{Z} H_y^{(s)}(x) = -H^{(i)} \quad \text{for} \quad \left| x \right| < \frac{w}{2}
\]
\[
H_z^{(s)}(x) = 0 \quad \text{for} \quad \left| x \right| > \frac{w}{2}
\]

where
\[
c(x) = \int_{-\infty}^{\infty} \frac{1}{\sqrt{1 - \mu^2}} \exp(ik_0 x \mu) d\mu = \pi J_0(k_0 x) + i Y_0(k_0 x)
\]

and \( w \) is the width of the nanogap or nanowire (Fig. 1). Here \( J_0(x) \) and \( Y_0(x) \) are Bessel functions of the first and second kinds of order zero, respectively. Considering that \( k_0 w \) will be very small, the Bessel functions can be approximated in terms of \( k_0 w \) and \( \ln(k_0 w) \). Further, using the average field values, \( E_{z0}(R_s) \) (gap) and \( H_{z0}(R_s) \) (wire) for the range \( \left| x \right| < w/2 \) in (8) and (9), respectively, we get

\[
E_{z0}(R_s) = 0 \quad H_{z0}(R_s) = 0
\]

expressions for electric [magnetic] field strengths for PEC (\( R_s = 0 \)) as
\[
E^{(i)} / E^{(0)} = \frac{E^{(0)}[-H^{(0)}]}{k_0 w[0.5 + \frac{1}{\pi}[\ln(k_0 w/4) - \gamma - 1]]}
\]

For cases of real metal (nonzero \( R_s \)), we also obtain
\[
\begin{align*}
E_{z0}(R_s) & \equiv \frac{E_{z0}(0)/E^{(i)}}{1 + 2R_s/Z} \\
H_{z0}(R_s) & \equiv \frac{H_{z0}(0)/H^{(i)}}{1 - 2(R_s/Z)H_{z0}(0)/H^{(i)}}
\end{align*}
\]
where $\gamma (= 0.577)$ is the Euler constant. Equation (10) shows $\sim 1/f$ as well as $\sim 1/w$ dependence of the field enhancement in the nanogap which agrees perfectly with the earlier experimental and numerical results [11].

Equations (10) and (11) above show that the field enhancement in the nanogap and nanowire remains the same for both the cases if the metals are PECs ($R_s = 0$). To verify this, we simulated the nanowire and nanogap structures using FDTD. The simulation result assuming perfect conductor has been plotted in Figs. 2(a) and 2(b) for thickness that is small enough ($t = 40 \text{ nm}$, $w = 1 \mu \text{m}$) so that the effect of finite thickness on field distribution is suppressed. To note, though the example considered here is from [11], our analysis applies equally well to other dimensions.

Now for the real metal, from (11) the behavior of field enhancement does not reduce as $R_s$ increases (real metal) because the resistivity of the metal $R_s$ is much smaller compared to $Z$ in general. However, for the nanowire, the product $R_s/Z$ in the denominator gets multiplied by $H_{\delta}(0)/H^{(i)}$ causing reduction in magnetic field enhancement as $R_s$ increases.

Equation (11) is not complete in the sense that the effects of finite thickness have been ignored. As the thickness increases, the conditions given in (4) and (5) are no longer valid. We name this as the geometric effect. This geometric effect can be accounted for in the analysis by assuming the current distribution to be essentially uniform within the wire, as long as $t$ is smaller than the skin depth $\delta$.

This small $t$ assumption also means that we are in the negligible retardation regime and the induced current in the nanowire only depends on $R_s$ and not on $t$ by itself. Under this condition, for the same amount of flux impinging on wires of identical widths $w$, the current $I$ will be constant given by $I = H_x(0, t)(2w + 2t) = H_x(R_w, 0)(2w)$. Now the expression for nanowire that takes into account the effect of finite thickness can be arrived at by multiplying Eq. (11) with $w/(w + t)$

$$H_x(R_w, t) = \frac{[H_x(0, 0)/H^{(i)}]w}{[1 - 2(R_z/Z)H_x(0, 0)/H^{(i)}](w + t)}.$$  

Arriving at the final equation for field enhancement including the geometric effect, we now study the field enhancement dependence on skin depth. Expressing field enhancement in terms of the skin depth (for the case of the nanowire) by writing $R_s = \rho/t$ [18] as $\delta^2 \omega \mu_0/2t$ (holds for low frequency, when $t < \delta$) in Eq. (12) where $\rho$ is the resistivity, we get

$$H_x(R_w, t) = \frac{H_x(0, 0)/H^{(i)} w}{1 + (\delta^2/\omega t) w} = \frac{H_x(0, 0)/H^{(i)} w}{1 + (\delta^2/\omega t) w + t}.$$  

$$b = \pi \left( \frac{\pi}{2} + \left[ \ln(k_0 w/4) - \gamma - 1 \right] \right).$$

The term $b$ above being of the order of 1, the magnetic field enhancement reduces when the product of $(w/\delta)$ and $(t/\delta)$ becomes smaller than 1. Figure 3 shows the magnetic field enhancement for different wire width and thickness. Perfect agreement is observed between calculated and numerically obtained results. For the electric field of nanogap, dependence on slit thickness was found to be negligible [Eq. (11) and FDTD data in Fig. 3(b)].

While the electric field enhancement of nanogap scales linearly with the reduction of gap width (even in the sub-skin-depth regime [11]), Eq. (13) and FDTD result in Fig. 3 above shows that the case is different for nanowire if $w/\delta$ or $t/\delta$ (dotted black lines in Fig. 3) is smaller than 1. Thus, contrary to the case of nanogap, an increase in the magnetic field enhancement of nanowire does not correlate well with the wire width but saturates in the sub-skin-depth regime, leading to a fundamental scaling length especially for magnetic field—the metal skin depth.

To put it intuitively, the outcome of the above analysis is that for the case of nanowire having finite conductivity, due to its boundary condition the magnetic field penetrates into the metallic region [Fig. 4(b)]. This means that the current

**FIG. 2 (color online).** Field enhancement obtained through FDTD analysis for $\lambda_0 = 3 \text{ mm}$. To bring out similar field pattern, we used PEC and thin ($t = 40 \text{ nm}$) metal structure. Width ($w$) for both structures is $1 \mu \text{m}$. (a) $E_x$ field enhancement at gap. (b) $H_x$ field enhancement at wire.

**FIG. 3 (color online).** (a) Magnetic field ($H$) enhancement for the nanowire plotted as a function of width: both analytical (lines) and FDTD analysis (+). (b) Field enhancement plotted as a function of $t$ for nanowire ($H$) and nanogap ($E$).
The flow is distributed within the metallic nanowire and hence, the resistance of the sub-skin-depth nanowire scales inversely with area [19] causing a saturation in the magnetic field enhancement. On the other hand for the case of nanogap, the field driven electric charges accumulate within the scale of the sub-nm Thomas-Fermi length at the gap surface [Fig. 4(a)], irrespective of the conductivity. Because of this charge accumulation on the “surface,” the effective width of the gap becomes independent of material conductivity as long as the gap width is considerably larger than sub-nm Thomas-Fermi length scales. Hence, in contrast to the nanowire, there is no conductivity dependent variation in the achieved electric field enhancement. To add, Fig. 5 shows the spatial distribution of the field enhancement at the exit ($y = 100$ nm) obtained from FDTD analysis for the nanogap and nanowire with same dimensions and material parameters as in Fig. 4.

To summarize, metallic nanowire structures (Babinet complementary structures for nanogaps) have been proposed and analyzed to achieve extraordinary magnetic field enhancement. Enhancement of the THz magnetic field by a factor of 300 (in terms of intensity, corresponds to a factor of 90 000) is obtained from the proposed nanowire. Analytic formulas derived incorporating thickness-normalized surface impedance concept shows that the achievable magnetic field enhancement in sub-skin-depth nanowires, though huge, cannot be as much as the electric field enhancement achieved in nanogaps. Orders of difference in the field enhancement were observed between nanogap and nanowire. More specifically, it was observed that when the cross sectional area of the nanowire becomes smaller than the square of skin depth of the material, there was a saturation in the achieved magnetic field enhancement. Our study provides detailed insights into the underlying physical phenomena that cause the deviations in classical Babinet’s principle. We also believe that our rigorous theory will work as valuable guidelines for designing efficient sub-skin-depth metallic components such as the nanowire and nanoresonator, suitable for future applications in magnetic sensors and magnetic nonlinearity based devices.

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