Performance Comparison of Delay-Interferometer Based Direct Detection oDOPSK Receivers

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Abstract: Performance evaluation of five different delay-interferometer based direct detection optical differential octal phase-shift keying receivers is reported. The performance is evaluated under an amplified spontaneous emission noise limited scenario using the well known Karhunen Loeve series expansion method. The classification of receivers into five different types is based on either the receiver schematic or the algorithm / methodology employed for symbol estimation.

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References and links

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1. Introduction

There has been increasing interest shown by the research community towards multilevel differential phase modulation schemes such as optical Differential Quadrature Phase Shift Keying (oDQPSK) and optical Differential Octal Phase Shift Keying (oDOPSK) [1-5]. As a consequence, several different proposals for oDOPSK receiver schematics with different levels of complexities and possibly different levels of performances have been reported [3-5].
To throw some insightful light on these receiver performances, Agrell and Karlsson [5] have reported a concise comparison of such receivers based on Euclidean distance considerations. There have also been suggestions of using error control coding schemes for performance enhancement of multilevel optical transmission [6-9]. To be more specific, there have been suggestions to use some memory based encoder at the transmitter side such as a convolutional encoder [6] or an encoder-modulation combination (coded modulation) as in [7,8]. The introduction of memory based encoding at the transmitter side dictates the usage of decoding mechanisms at the receiver side which estimates transmitted symbol sequences with the help of maximum likelihood sequence estimation algorithms like the Viterbi algorithm [10]. Also, the performance improvement methods suggested in [7-9] make use of the soft detection approach wherein the decision on the transmitted symbol is taken by the receiver based ideally on unquantized analog outputs from the detector.

In the light of this, it would be of importance to see how the different receiver schematics proposed for oDOPSK would work on the principles of maximum likelihood symbol estimation with soft unquantized outputs. It is to be noted that in direct detection optical communication systems such as those under investigation in this paper, the noise statistics is non Gaussian [1, 11]. The maximum likelihood detection criteria turn into a minimum Euclidean distance measure with certainty only when the noise is Gaussian [12]. However, the Euclidean distance could still be used as a ‘rule of thumb’ to make a quick assessment of receiver performances as has been reported in [5]. The aim of this paper is thus to fill the gaps, if any, in the concise analysis presented in [5] due to the non-Gaussian nature of noise and to extend the analysis presented therein to a more comprehensive set of detection schemes and receiver schematics.

This paper analyzes five different oDOPSK receivers and compares their performances. We use the Karhunen Loeve series expansion (KLSE) based method [13,14] and thereby take into account the actual noise distribution to evaluate the receiver symbol error rate (SER) in an amplified spontaneous emission (ASE) noise limited scenario. To avoid digression, the transmission medium is assumed to be free of nonlinearities and dispersion. Some possible receiver selection criteria in light of the SER performances, hardware complexities and applications are as well discussed.

2. Theoretical comparison of electrical SNR performances

A comparison of the electrical signal to noise ratio (SNR) performance is reported in this section. The electrical SNR analysis based on Euclidean distance as reported in this section is to arrive at a quick assessment of the various receiver schematics considered in this paper. A more rigorous performance evaluation based on the semi-analytic KLSE method that takes into account the actual noise distribution is reported in the next section.

As is evident from [12], in order to perform the maximum likelihood symbol estimation, an exact knowledge of the noise statistics is necessary. While the ASE noise is Gaussian, the squaring operation at the photo-detector combined with any post-detection electrical filtering alters the noise statistics from Gaussian to non Gaussian [1, 11]. Exact analytical expression for the characteristic function (CF) of the post-detection electrical filter output is obtainable using the KLSE method discussed in [13, 14]. It is also possible to obtain an expression for the probability density function (pdf) through numerical integration and curve fitting subroutines. However, the above in combination with formulation and implementation of a fast algorithm for the receiver to perform the required computations for maximum likelihood estimation can be a clumsy exercise. The required electronic complexity for the above operations to be performed in real time could also be prohibitive. On the other hand, one can proceed under the assumption that the maximum likelihood estimation is equivalent to minimum Euclidean distance comparison. Even with the state-of-the-art electronic hardware technology, the former seems less feasible and hence, in the following, we proceed with the performance evaluation assuming that maximum likelihood estimation process is minimum Euclidean distance comparison process. We discuss more on this later in section 3.
It is also to be noted that the received noisy vector is used ‘as it is’ in its analog form (or its digital approximation using sufficiently small quantization step) in the above discussed maximum likelihood symbol estimation process. Since the received vector is used ‘as it is’, we call this detection scheme as ‘soft-detection’. On the other hand, if the transmitted symbol is estimated after comparing the components of the received vector with some threshold (two-level quantization), the detection scheme is identified as ‘hard-detection’.

The types of receivers compared are as follows

**Type-1**: A two delay-interferometer receiver as shown in Fig. 1 and based on soft detection.

**Type-2**: A four delay-interferometer receiver as shown in Fig. 2 and as in [3] which estimates the symbol through direct binary decisions on the four parallel outputs - hard detection.

**Type-3**: Same as Type-2 except that the symbol is estimated based on soft detection.

**Type-4**: The two delay interferometer receiver discussed in [3] that employs hard detection and which has a similar structure as that shown in Fig. 1.

**Type-5**: A two delay interferometer receiver proposed in [4] and as shown in Fig. 3, which incidentally can only be of use in hard detection.

At such values of SNR maintained in practice, the probability of a symbol being mistaken as one nearest to it in the signal space diagram will be considerably more than the probability of it being mistaken for symbols that are farther away from it [12]. Thus, for the following electrical SNR analysis, we use the assumption that the SER is dominated by the minimum Euclidean distance error event [12].

For the receiver **Type-1**, having received a vector \( \mathbf{r} \) corrupted by noise, a decision among symbols \( s \) and \( k \) is made by the receiver based on a Euclidean distance comparison between \((\mathbf{r}, s)\) and \((\mathbf{r}, k)\) and the transmitted symbol is estimated as the one nearer to \( \mathbf{r} \) [12]. Assuming without loss of generality that \( s \) was transmitted, an error in estimation in the present case can be expressed as

\[
\sum_{i=1}^{2} (s_i - k_i) r_i < 0 \tag{1}
\]

Here \((s_1, s_2), (k_1, k_2)\) and \((r_1, r_2)\) correspond to the \((y_1, y_2)\) coordinates of \( s, k \), and \( r \) respectively. The electrical SNR would be then the mean square of the term on the left hand side of (1) divided by its variance as given below.

\[
\text{SNR}_{\text{Type-1}} = d_{\text{in}}^2 / (4\sigma^2) \tag{2}
\]

Here \( \sigma^2 \) is the variance of each of the sampled outputs from the delay-interferometer and \( d_{\text{in}} \) is the minimum Euclidean distance as shown in the signal space diagram of Fig. 1(b).

For the receiver **Type-2**, from the analysis presented in [3], it can be deduced that the electrical SNR would be equal to the one given in (2) above. For **Type-3** receiver, a close look reveals that it comprises of two **Type-1** receivers in parallel with output pairs \( y_1, y_3 \) and \( y_2, y_4 \) respectively. To analyze the receiver performance under the soft detection assumption, we plot two 2 dimensional signal space diagrams with a relative offset of \( \pi/4 \) radians between them, as shown in Fig. 2(b) and 2(c).

Having received a noisy vector \( \mathbf{r} \), following the discussion presented for receiver **Type-1**, the minimum distance error event in this case can be expressed as,

\[
\sum_{i=1}^{4} (s_i - k_i) r_i < 0 \tag{3}
\]

requiring summation over four terms due to the four outputs from the receiver. The variance \( \sigma_2^2 \) of the term on the left side of (3) can be readily derived by taking note of the fact that for **Type-3** receiver, the correlation between the outputs due to the phase differences between the interferometers not being orthogonal comes into prominence [11]. Taking into account this correlation, the variance can be shown to be...
\[ \sigma^2_V = (d_m^2 \sigma^2 / 8) \left[ 1 + \sum_{i=1}^{r} \sum_{j=1}^{i} (s_i - k_i)(s_j - k_j) \rho_{ij} / d_m^2 \right] \]  

(4)

where \( \rho_{ij} \) is the correlation coefficient defined as

\[ \rho_{ij} = \langle (r_i r_j^*) - \langle r_i \rangle \langle r_j^* \rangle \rangle / \sigma_m^2 / 4 \]  

(5)

Following the theoretical treatment presented in [11], the correlation coefficients \( \rho_{ij} \) can be shown to be

\[ \rho_{12} = 1/\sqrt{2}, \ \rho_{31} = 0, \ \rho_{24} = -1/\sqrt{2}, \ \rho_{23} = 1/\sqrt{2}, \ \rho_{24} = 0, \ \rho_{44} = 1/\sqrt{2} \]  

(6)

Now using (6) in (4) gives the variance \( \sigma^2_V \) as \( \sigma^2_m^2 / 4 \) and dividing the square of the mean of the left hand side of (3) with this variance, we obtain the SNR as

\[ \text{SNR}_{Type-4} = d_m^2 / (4 \sigma^2) \]  

(7)

With regards to the Type-4 receiver, it can be deduced from [3] that the dominant error event would constitute an erroneous decision among the projection of any two symbols that lie in the same quadrant on to the signal space axes (Fig. 1(b)). Hence the minimum distance would be \( d_m / \sqrt{2} \) and not \( d_m \). This will result in a 3dB inferior electrical SNR compared to that in (2) and (7) and as discussed in [3] and [5].

Finally, regarding receiver Type-5, we note that the outputs \( y_1 \) and \( y_2 \) identify the quadrant in which the transmitted symbol lies and the discrimination among the two symbols that lie within that quadrant is done by considering the difference between the absolute values of \( y_1 \) and \( y_2 \). Probability of a decision error in this receiver schematic could be identified as the

Fig. 1 (a) Type-1 receiver schematic (b) Signal space diagram; \( T \) stands for a delay equivalent to one symbol duration

Fig. 2 (a) Type-2 receiver schematic (b) Signal space diagram with respect to \( y_1 \) and \( y_3 \) (c) Signal space diagram with respect to \( y_2 \) and \( y_4 \); \( T \) stands for a delay equivalent to one symbol duration

Fig. 3 Type-5 receiver schematic [4]; \( T \) stands for a delay equivalent to one symbol duration.
probability of the first two events.

1) The received vector landing in the wrong quadrant or
2) The received vector landing in the right quadrant but the decision among the two symbols in a quadrant going wrong.

At such values of optical SNR (OSNR) at which a communication system operates, the probability of the first event will be dominated by the probability of mistaking the quadrant as the one nearest to the actual transmitted symbol. The second event occurs when only one of the two binary decisions, i.e. that on \( y_1 + y_2 \) or \( y_1 - y_2 \) goes wrong.

Without loss of generality, considering that the transmitted symbol was in first quadrant that is nearer to the \( y_1 \) axis, the first and second event will be dominated by the following events expressed as \( r_2 < 0 \) and \( r_1 - r_2 < 0 \) respectively with \( r_1 \) and \( r_2 \) corresponding to the outputs \( y_1 \) and \( y_2 \). The variance of \( r_1 \) and \( r_1 - r_2 \) will be \( \sigma^2 \) and \( 2\sigma^2 \) respectively. Further, the square of the mean of \( r_1 \) and \( r_1 - r_2 \) can be readily computed as \( \frac{d^2}{2} \) and \( \frac{d^2}{2} \) respectively; which leads to the SNR related to both the events \( r_2 < 0 \) and \( r_1 - r_2 < 0 \) as

\[
\text{SNR}_{	ext{type}, 5} = \frac{d^2}{4\sigma^2} 
\]

Table 1 below summarizes the results of the above SNR analysis.

<table>
<thead>
<tr>
<th>Type</th>
<th>Detection Method</th>
<th>No. of Detectors</th>
<th>Post-Detection Electronics</th>
<th>SNR</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Soft</td>
<td>2</td>
<td>Complex</td>
<td>( \frac{d^2}{4\sigma^2} )</td>
</tr>
<tr>
<td>2</td>
<td>Hard</td>
<td>4</td>
<td>Moderate</td>
<td>( \frac{d^2}{4\sigma^2} )</td>
</tr>
<tr>
<td>3</td>
<td>Soft</td>
<td>4</td>
<td>Complex</td>
<td>( \frac{d^2}{4\sigma^2} )</td>
</tr>
<tr>
<td>4</td>
<td>Hard</td>
<td>2</td>
<td>Simple</td>
<td>( \frac{d^2}{8\sigma^2} )</td>
</tr>
<tr>
<td>5</td>
<td>Hard</td>
<td>2</td>
<td>Moderate</td>
<td>( \frac{d^2}{4\sigma^2} )</td>
</tr>
</tbody>
</table>

3. Numerical evaluation of SER / discussions

As mentioned in the previous section, we consider that the SER is dominated by the minimum Euclidean distance error event. The SER analysis for the five different receiver schematics were conducted using the KLSE based method reported in [13, 14], which takes into account the exact pdf of the noise. The bit rate was fixed as 40Gbps for all the receiver types and a pseudo random binary sequence (PRBS) of length \( 2^{29}-1 \) was used for the semi-analytic SER evaluation [3]. Return-to-zero signaling format was employed and the OSNR was calculated with unpolarized ASE within a reference bandwidth of 0.1 nm. The optical bandpass filter was modeled as a first order Gaussian filter with its 3 dB bandwidth as equal to three times the signaling rate while the postdetection electrical filter was modeled as a fifth order Bessel filter with its 3 dB bandwidth as equal to the signaling rate.

For the receivers Type-1 and Type-3, since the CF of the post-detection electrical filter output is known from the KLSE based method, the probability of the event reported through equations (1) and (3) could be evaluated for each of the transmitted symbols. The average of such probabilities over the symbols transmitted would provide the estimate of SER. For receivers Type-2 and 4, the procedure reported in [3] could be used for evaluating the SER.

For Type-5 receiver, for each of the transmitted symbols, with an exact knowledge of the CF of the post-detection electrical filter output provided by the KLSE based method, the probability of events 1) and 2) described in the previous section could be evaluated. The average of such probabilities over the symbols transmitted provides an estimate of the SER.

The SER curves for the five different receiver schematics as function of OSNR are plotted in Fig. 4. It can be seen that the SER curves of all receivers except Type-4 are more or less coinciding with each other. The Type-4 receiver exhibits 3dB poorer OSNR sensitivity compared to the rest, as hinted in earlier theoretical discussion. Also, the congruence between
the SER curves of Type-1 and Type-2 receivers goes on to validate the comment by Agrell and Karlsson [5] that the receiver Type-2 as proposed in [3] is in principle equivalent to a maximum likelihood detection receiver and is optimum in that sense.

Even though the results shown in Fig 4. have been computed under the possibly ‘suboptimum’ assumption that the maximum likelihood estimation is equivalent to the minimum Euclidean distance comparison, the computations have been performed using the exact CF of the post-detection electrical filter output. Thus, no Gaussian pdf assumption was made for the post-detection electrical filter outputs while computing the SER. This gives the actual SER performance of the compared receivers under a detection scheme where the symbol is estimated as the one which is closest to the received vector in terms of Euclidean distance. Moreover, since the same assumptions have been applied to all the receivers compared, even if a performance comparison is conducted under an exact maximum likelihood estimation scenario, it cannot so happen that the comparative nature of the results reported herein would be grossly different. In other words, the relative difference among the receiver performance curves would remain the same as given herein even if a performance comparison is conducted under an exact maximum likelihood estimation scenario.

Based on these results, we comment as follows. As far as SER performance alone is concerned, all the receivers except Type-4 are equivalent. However, the following considerations can play a role in the selection of receivers for different applications. Type-5 receiver needs only two interferometers but has to have an electronic system to add / subtract the two outputs from the interferometers. Additionally required processes are binary decisions followed by an XOR operation. While there is also an equivalent four delay interferometer version of Type-5 receiver [4] with *slightly lesser* demands on electronic hardware, the advantage of having to use only two delay interferometers / detector pairs is lost. The Type-2 receiver also requires some post detection logical operations and the electronic hardware complexity is comparatively the same as that of Type-5. Thus, both Type-2 as well as Type-5 receiver need some varying degree of optical complexities / post detection electronic logic operations that have to work at speeds corresponding to the signaling rate.

In such situations where maximum likelihood estimation of transmitted sequences become *imperative* [7-9], the choice of a receiver has to be between Type-1 and Type-3. Between these, since they both have identical performance due to the correlation among the outputs in Type-3 receiver, the obvious choice would be Type-1 since it needs only two delay interferometers / detector pairs. This was indeed the receiver schematic used in [7] and [9].

4. Conclusions

Performances of five different types of direct detection oDPSK receivers were compared. It was observed that for applications involving maximum likelihood sequence estimation, the receiver schematic that uses two delay interferometers (Type-1) was the best. For applications which do not require soft decoding schemes, we confirm that Type-2 is the optimum choice as suggested in [5], or equivalently, Type-5 structure - subject to electronic hardware demands.