Semi-empirical multi-port lattice model for long-period fiber grating analysis under arbitrary temperature distributions

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Abstract: We propose a semi-empirical model for the complete analysis (spectrum, bandwidth, and wavelength / phase shifts) of a temperature-tuned long-period fiber grating (LPFG) filter. By applying the multi-port lattice model to LPFGs, while deriving and utilizing the empirically determined temperature-dependence of core-to-cladding intermodal dispersions, we achieve a precise, practical means of spectrum analysis. Excellent agreement of the model with the experimental results was obtained over wide spectral ranges.

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OCIS codes: (060.2330) Fiber optics communications; (350.2770) Gratings.

References and links

1. Introduction

Long-period fiber gratings (LPFGs) have been a topic of continuous research interest, with importance in practical applications [1-4] such as; band-rejection filters, optical fiber polarizer, sensors, and optical switches. With their low insertion-loss / back-reflection and polarization insensitivity [5], LPFGs also provide an attractive solution to the gain equalization problem of fiber amplifiers. In this regard, various approaches have been suggested to custom engineer the spectrum of an LPFG filter to that of the gain-flattened optical amplifier, for example concatenated LPFGs [5], phase-shifted LPFGs [6] or non-uniform LPFGs [7]. The tuning of the LPFG spectrum has also been achieved by adopting various control techniques, including the control of the temperature [8, 9] or strain [3] over the grating (whole or part of), with proper distribution functions. For all of these applications, it is crucial to be able to precisely understand, predict and engineer, at will, the spectral response of the LPFG - as well having a method of analysis for the evolutions / couplings of the propagation modes.

To provide a reliable and convenient tool supporting LPFG analysis, various approaches have been suggested in the past, including spectral summation [5] and fundamental matrix models [6,7]. Further elaboration of the LPFG model was also achieved recently, including: 1) the multi-port lattice model [10] for the analysis of multiple cladding modes, and 2) the extended fundamental matrix model (considering only one cladding mode) for the inclusion of a temperature dependent mode coupling term in the analysis [11]. Nevertheless, as regards the practical application of these techniques, there has not been a complete and simple method of non-uniform LPFG analysis enabling the estimation of the spectral characteristics under arbitrary temperature distributions, while accommodating the interactions of multiple cladding modes. Employing - for the determination of the inter-modal dispersion relations - either full modal analysis [12] requiring a full series of detailed, precise fiber parameters, or the high-order fitting method [11] with its fitting function limited to the peak wavelength while neglecting phase relations, these approaches also suffer from inherent limitations, often failing to derive the exact filter spectral shape (especially for asymmetric filters, e.g. for [11]) or leading to erroneous results from the imperfectly assumed fiber parameters (note that as many as several 10’s of related fiber parameters have to be used to calculate the precise dispersion relations with the full-modal analysis).

In this paper, we develop a simplified form of the multi-port lattice model by introducing semi-empirical mode-coupling functionals, in order to provide a convenient means of analysis for a general, multiple grating LPFG, having multiple cladding modes under arbitrary temperature distributions. By using the empirically determined intermodal dispersion functions, we achieve a precise, complete and convenient method of spectral shape analysis, allowing for the prediction of the transmission spectra, wavelength shift, bandwidth / phase changes of the grating over wide spectral ranges (1000-1700nm). The comparison of the model to the experimental results obtained with a temperature-tuned (independently controlled, 64-segment coil heater) LPFG shows excellent agreement in terms of its transmission spectrum.

2. Formulation of the principle

Keeping in mind that the transmission spectrum of the LPFG is determined by the interaction between the core mode and multiple cladding modes, without any loss of generality we start our discussion with the $2 \times 2$ fundamental matrix model describing the interaction (intermodal dispersion) between the core mode (0) and one of the cladding modes (p). Focusing on the temperature dependences of the key parameters in the LPFG analysis, viz. the detuning factor $\delta(\lambda, T)$ and coupling coefficient $\kappa(\lambda, T)$ of the cladding mode, we first focus our attention on $\kappa$. Employing the notation in [3, 11] and noting that the temperature dependencies for $\kappa$ can be ignored for all practical purposes (reported to be <1% [11] - mostly depending on the dielectric structure of the fiber [12]) we will assume the following expression of $\kappa$ for all later discussions,
\[ \kappa(\lambda, T) = \frac{2\pi\sigma}{\lambda} \cdot C(\lambda, T) = \frac{2\pi\sigma}{\lambda} \cdot C(\lambda) \]  
(1)

where \( \sigma \) is the normalized induced-index change, and \( C(\lambda, T) \) is the overlap integral between the core and cladding mode filed over the fiber cross section[10], [12]. Next, for the convenient handling / manipulation of the detuning factor \( \delta(\lambda, T) \), without any loss of generality we now employ the expression in [3] and then rewrite the equation - in terms of the detuning factor at fixed temperature \( \delta_0(\lambda, T_0) \), and the differential dispersion function \( \Delta \Phi(\lambda, T-T_0) \) - as follows,

\[ \delta(\lambda, T) = \frac{1}{2} \left( \beta^{(0)} - \beta^{(p)} - \frac{2\pi}{\Lambda} \right) = \frac{1}{2} \left( \Phi(\lambda, T) - \frac{2\pi}{\Lambda} \right) = \frac{1}{2} \left( \Phi(\lambda, T) + \Delta \Phi(\lambda, T-T_0) - \frac{2\pi}{\Lambda} \right) = \delta_0(\lambda, T_0) + \frac{\Delta \Phi(\lambda, T-T_0)}{2} \]  
(2)

where \( \Phi \) is the intermodal (core-to-cladding) dispersion function, \( \Lambda \) the pitch of the grating, \( \beta \) the propagation constant of the corresponding modes, and \( \lambda \) the vacuum wavelength. Worth to note, the temperature dependence of grating period \( \Lambda \) was ignored considering the order of thermal expansion coefficients (0.55 \( \times \) 10^{-6}/°C) of silica-based single mode fiber [13].

Meanwhile, also utilizing the definition of \( \Phi(\lambda, T) = \beta^{(0)} - \beta^{(p)} \), we note that \( \Delta \Phi(\lambda, T-T_0) \) can be alternatively expressed in terms of the effective index of the core and cladding modes.

\[ \Delta \Phi(\lambda, T-T_0) = \frac{2\pi}{\Lambda} \left( \Delta n^{(0)}_c(\lambda, T-T_0) - \Delta n^{(p)}_c(\lambda, T-T_0) \right) \]  
(3)

Now, in order to express the above Eq. (3) in terms of empirically accessible parameters, we use the well-known formula for the normalized effective index of core mode, \( b \) [14],

\[ b = \frac{n^{(0)}_c}{n^{(0)}_c - n^{(p)}_c} \equiv \left( 1.1428 - \frac{0.9960}{V} \right)^2 \]  
(4)

expressed in terms of the normalized frequency, \( V \) [14],

\[ V = \frac{2\pi n}{\lambda} \cdot \sqrt{n^2 - n^2_j} \]  
(5)

to write \( n^{(0)}_c = b(n^{(0)}_c - n^{(p)}_c) + n^{(0)}_c \), and then \( \Delta n^{(0)}_c = \Delta \left[ b(n^{(0)}_c - n^{(p)}_c) + n^{(0)}_c \right] \), where \( a \) is the core radius.

Now, rewriting \( \Delta \Phi(\lambda, T-T_0) \) in Eq. (3) in terms of \( b \), and its differential, \( \Delta b \), we get,

\[ \Delta \Phi = \frac{2\pi}{\Lambda} \left( \Delta n^{(0)}_c - \Delta n^{(p)}_c \right) = \frac{2\pi}{\Lambda} \left[ \Delta \left( n^{(0)}_c - n^{(p)}_c \right) + b \Delta \left( n^{(0)}_c - n^{(p)}_c \right) + \Delta n^{(0)}_c - \Delta n^{(p)}_c \right] \]  
(6)

Further simplification of the above expression can be accomplished in a straightforward manner from the definition of \( V \) and also by using (4) for the calculation of \( \Delta V \) and \( \Delta b \);

\[ \Delta V = \frac{2\pi n}{\lambda} \cdot \frac{n \Delta n^{(0)}_c - n \Delta n^{(p)}_c}{\sqrt{n^2 - n_j^2}} \equiv V \cdot \frac{\Delta n^{(0)}_c}{2(n^{(0)}_c - n^{(p)}_c)} \quad ; \quad n^{(0)}_c = n^{(p)} \]  
(7)

\[ \Delta b = 2 \left( 1.1428 - \frac{0.9960}{V} \right) \cdot \frac{0.9960}{V^2} \quad \Delta V = \left[ 1.1428 - \frac{0.9960}{V} - \left( \frac{0.9960}{V} \right)^2 \right] \left( \frac{\Delta n^{(0)}_c - n^{(p)}_c}{n^{(0)}_c - n^{(p)}_c} \right) \]  
(8)

Finally, with \( V = 2.405 \frac{\lambda_{\text{cutoff}, T_0}}{\lambda} \) and using \( \Delta n^{(p)}_c = \Delta n^{(p)}_c \) (as all the cladding modes ~ up to 20th - are confined almost entirely within the cladding material), we derive the interim expression of \( \Delta \Phi(\lambda, T-T_0) \), which can be used in Eq. (2); expressed as the product of the wavelength dependent part and temperature dependent component,

\[ \Delta \Phi(\lambda, T-T_0) \equiv \frac{2\pi}{\Lambda} \left( 1.1428 - \frac{1.1428 \cdot 0.9960}{2.405 \cdot \lambda_{\text{cutoff}, T_0}} \right) \Delta (n^{(0)}_c - n^{(p)}_c) \]  
(9)
where $\lambda_{\text{cutoff},T_0}$ is the cutoff wavelength of the fiber at fixed temperature.

With the above expression in Eq. (9), the LPFG analysis problem reduces to a much simpler form, requiring the empirical characterization of $\Delta(n_{\text{co}}-n_{\text{cl}})$ as a function of temperature. For this purpose, we first obtain $\Phi(\lambda,T)$ and its differential, $\Delta \Phi(\lambda,T)$, from the measurement, and then use the fitting Eq. (9) to get $\Delta(n_{\text{co}}-n_{\text{cl}})$.

3. Determination of parameters

![Intermodal dispersion plot](image1)

Fig. 1. Intermodal dispersion $\Phi(\lambda,T)$ for 1st ~ 7th cladding modes extrapolated from experimental data, utilizing phase matching condition [3] and resonance peak wavelength of each grating.

![Experimental values plot](image2)

Fig. 2. Experimentally obtained (a) difference of intermodal dispersion $\Delta \Phi(\lambda,T-T_0)$ and (b) core-to-cladding refractive index difference $\Delta(n_{\text{co}}-n_{\text{cl}})$ values at different temperatures (reference temperature = 25°C, 80°C, 140°C), for 1st to 7th cladding modes.

Figure 1 shows the plot of the $\Phi(\lambda,T)$ values obtained from the spectrum measurement on the different LPFGs (with $\Lambda=625, 540, 496, 440, 400$ and 344um for the 1st to 6th cladding modes and $\Lambda=344, 348, 352, 356,$ and 360um for the 7th cladding mode. $T$ was varied between 26 and 140°C. The $\Phi(\lambda,T)$ values were calculated from the measured peak wavelengths and grating period, $\Lambda$, at each $\lambda$ and $T$ [3]). Figure 2 also shows the plot of the $\Delta \Phi(\lambda,T-T_0)$ values.
derived from Fig. 1 for the 1st ~ 7th cladding modes (wavelengths between 1000nm to 1650nm). It is worth mentioning that the measured $\Delta \Phi(\lambda, T-T_0)$ and $\Delta(n_{co}-n_{cl})$ values for the different cladding modes overlap in a perfect manner, consistent with the result based on the full-model analysis [8].

In this regard, it can be said that the temperature variation of the core-to-cladding index difference is indeed the dominant fundamental parameter in the determination of the temperature dependence of the intermodal dispersion, irrespective of the order of the cladding modes, implying that one does not have to experimentally characterize $\Delta \Phi(\lambda, T-T_0)$ for each and every cladding mode. Especially, by taking advantage of the linear relationship observed in Fig. 2(b) between $\Delta(n_{co}-n_{cl})$ and the temperature variation, one can rewrite the Eq. (9), at least within the tuning range of interest (25 ~140°C), as follows;

$$\delta(\lambda, T) \equiv \delta\lambda(\lambda, T_0) + \frac{\pi}{\lambda} \left(1.1428^2 - \frac{1.1428 \cdot 0.9960}{2.405 \cdot \lambda_{cutoff}^2} \lambda\right) \Delta(n_{co}-n_{cl})$$

$$\equiv \delta\lambda(\lambda, T_0) + \frac{4.103}{\lambda} \frac{1.487 \lambda_{cutoff}}{\lambda_{cutoff}^2} \alpha_T \cdot (T-T_0)$$

with a cutoff wavelength of the fiber $\lambda_{cutoff}$ at fixed temperature, and a fitting constant $\alpha_T$, which can be determined from simple measurements of the uniform LPFG spectrum at several different temperatures (focusing on only one of the cladding modes).

4. Application of the semi-empirical LPFG analysis model

Figure 3 shows the experimental setup employed to synthesize the filter spectrum, used for the verification of the LPFG analysis method based on our semi-empirical construction. Tunable LPFG with 64 sections of independent thermal controllers (coil heaters for temperature tuning between 26 to 126°C) were used [9]. With the above experimental setup, independent controls of the coupling properties at different positions of the non-uniform LPFG sections were possible. It is worth mentioning that the fiber used in the LPFG construction was Fibercore photosensitive fiber (Boron co-doped Ge-Si fiber) with a cutoff wavelength $\lambda_{cutoff} = 1153\text{nm}$. For this specific fiber, the measured value of $\alpha_T$ was $1.66 \times 10^{-6}/\text{°C}$.

For this analysis, we use the multiport lattice filter model [10] to simultaneously consider the couplings of multiple cladding modes over the whole bandwidth of interest (note that conventional 2x2 models with core mode + 1 cladding mode are limited in their wavelength range or in the treatment of the concatenated LPFGs with different grating properties having multiple cladding modes in the same wavelength range). Dividing the general non-uniform LPFG with arbitrary temperature distributions into $k$ uniform LPFG sections, we now build the transfer matrix $Q$ in (11) from the product of the $M_k(T_k)$’s, viz. the transfer matrix of the uniform LPFGs at temperature $T_k$, with its elements calculated from (1) and (7), as follows:

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#89780 - $15.00 USD  Received 14 Nov 2007; revised 28 Dec 2007; accepted 3 Jan 2008; published 8 Jan 2008
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where \( E^{(0)}(L) \) and \( E^{(0)}(L) \) are the resulting electric fields in the core and cladding, respectively.

### 4.1 Example 1

As a first application example, we tested a cascaded LPFG composed of two uniform LPFGs with different grating periods \((\Lambda_1=348 \mu m \text{ and } \Lambda_2=360 \mu m \text{ for LPFG1 and LPFG2, respectively, as shown in Fig. 4})\), but utilizing the same cladding mode \((7^{th})\). For the prediction of the LPFG spectral shape, given the inherent limitations in the conventional, lightweight approaches such as the \(2 \times 2\) matrix model or higher order fitting method, one can employ the full modal analysis [12] with a series of precisely characterized fiber parameters or alternatively the proposed semi-empirical analysis method whose results are shown in detail below.

For the analysis, the wavelength dependence of the dispersion function \( \Phi (\lambda, T_0) \) obtained in the previous section was used. With the wavelength dependent curvature of the intermodal dispersion function of the \(7^{th}\) cladding mode \( \Phi^{(7)} \) (Fig. 5(b), data from Fig. 1), the intermodal dispersion comes to have a different slope for the different grating periods, \( \Lambda_1 \) and \( \Lambda_2 \), resulting in different spectral shapes at \( \lambda_{1\text{ res}} \) and \( \lambda_{2\text{ res}} \). Utilizing Eqs. (1) and (10) to get \( \kappa \) and \( \delta \), respectively, and then using (11) to get the final spectrum, the analytically obtained spectrum peak wavelength \( \lambda_{\text{res}} \) and FWHM (full width half maximum) bandwidth \( \Delta\lambda_{\text{FWHM}} \) were 1506nm and 46nm for LPFG1, and 1628.4nm and 25nm for LPFG2, respectively, exactly overlapping with the experimentally determined values [Fig. 5(a)].
4.2 Example 2

To examine the spectral changes of the LPFG under temperature distribution control, a cascaded non-uniform LPFG composed of three uniform LPFGs was used (Fig. 6 grating period $\Lambda_1=\Lambda_3=356\mu m$, $\Lambda_2=433\mu m$). For the analysis, the dispersion function $\Phi(\lambda, T_0)$ and its temperature differential $\Delta\Phi(T-T_0)$ obtained in the previous section were used. Since the core mode of LPFG1 / LPFG3 mainly couples with the 7th cladding mode, and the core mode of LPFG2 couples with the 6th cladding mode, all the other cladding modes were ignored in the calculation within the wavelength range of interest.

Fig. 6. Cascaded LPFGs with different cladding modes.

Fig. 7. (a), (c), (e) Measured / calculated LPFG spectra and (b), (d), (f) variations of intermodal dispersion under various temperature distributions (inset).
In Figs. 7(a) and 7(b), we show the transmission spectra and intermodal dispersion of the LPFG held at room temperature (26°C, see the inset of the figure). Since LPFG1 and LPFG3 have identical grating properties (length and period), the coupling of LPFG1’s 7th cladding mode to LPFG3’s 7th cladding mode was observed, exhibiting interference fringes within the resonance band of the composite grating (with the free spectral range of the comb $F_{\text{FSR}} = 12\text{nm}$). Figure 7(c) also shows the spectrum of the LPFG at a temperature of 120°C. Not only a shift in the resonance wavelength, but also changes in $F_{\text{FSR}}$ of the 7th cladding mode (12nm to 9nm) were predicted from the model and were confirmed by experiment. It should be mentioned that the changes in $F_{\text{FSR}}$ are mainly due to the temperature induced vertical shift in the dispersion curve $\Phi^{(7)}$ (for LPFGs 1 and 3), and corresponding changes in the intermodal dispersion slope at the resonance frequency. It is also worth noting that even with almost identical $\Delta\Phi$ or $\Delta(n_{\text{co}}-n_{\text{cl}})$ values for the 6th and 7th cladding modes, the resonance wavelength shift of the 7th cladding mode is much larger than that of the 6th cladding mode, because of the steeper intermodal dispersion slope of the former.

Finally, setting the temperature of LPFG2 to 126°C (LPFG1, 3 at 26°C), a blue shift in the resonance peak $\lambda_{2\,\text{res}}$ of LPFG2’s 6th cladding mode was observed which, it is important to note, was accompanied by spectral changes of the 7th cladding mode [Fig. 7(e)]. We attribute this to the temperature-induced phase changes in LPFG2 affecting the coupling of the 7th cladding modes between LPFG1 and LPFG3 which, although minor, led to changes in the resonance peaks and spectral profiles. It is important to note that the above example clearly demonstrates the successful treatment of multiple cladding modes and the couplings in-between them using our simplistic, semi-empirical format. Accurate analysis on the temperature dependencies of LPFGs were also achieved (which has not been practically feasible with the full-modal analysis or high-order fitting method) for our approach, using external parameter sets measured with minimal efforts.

4.3 Example 3

![LPFG unit for tunable EDFA gain equalization filter; cascaded LPFGs with different grating lengths and different grating periods.](image)

As a final application example, we analyze the dynamic spectrum control of the EDFA gain equalization filter. For this purpose, two concatenated, thermally controlled LPFGs were used. The grating period and length were 356μm and 11cm for LPFG1, and 433μm and 5cm for LPFG2, respectively.

By independently applying the currents to each of the 64 segments of the coil heaters distributed over the whole grating, piecewise-uniform control / tuning of the LPFG temperature distribution was achieved. Figure 9 shows the measured (pink dotted line), and theoretically calculated spectrum curves (black line). The initial LPFG spectra [at room temperature, 28°C, in Fig. 9(a)] are shown in Fig. 9(b). Also shown in the figure are the temperature distributions and achieved (experimental, theoretical) optimized transmission spectra of the LPFGs for the EDFA ASE spectrum, in the case where its pump is driven with currents of 51mA and 71mA. Excellent agreement in the transmission spectrum was observed, revealing the contributions of each cladding mode (6th: dash, 7th: dot).
Fig. 9. LPFG based dynamic gain equalization filter; Applied temperature distributions (a, c, e) and their LPFG spectra (b, d, f).

6. Conclusion

We proposed and experimentally verified a simplified method of LPFG analysis based on a semi-empirical formulation. Empirically determined intermodal dispersion functions were introduced and used to handle the spectral shape of non-uniform gratings possessing arbitrary temperature distributions. The application of our empirical formulation to a multi-port lattice matrix framework allowed for the successful treatment of multiple cladding modes and the couplings in-between them to be achieved. The analysis on the transmission spectra of general, multiple-cladding mode LPFGs under an arbitrary temperature distribution and grating periods was demonstrated, without relying on the rigorous full model analysis or higher-order fitting methods. Excellent agreement with the experimental results was observed in the analysis of the peak wavelength shape/shift, phase difference and FSR variations. Given its simple and general formulation based on the key fundamental fiber characteristics (temperature variations of the core-to-cladding index difference, thus requiring the characterization of the LPFG at only a few subsets of the cladding modes), we believe that our approach offers an excellent but convenient means of general, dynamic LPFG analysis.